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Large Scale Structure Formation in the Universe

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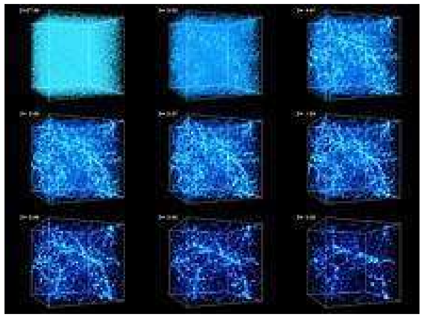
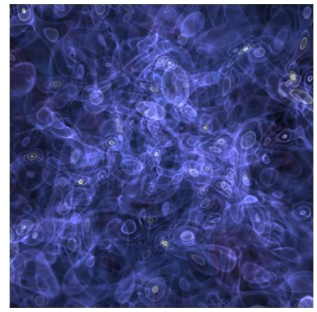
1 Introduction





Humanity has been mesmerized by stars for as long as we can remember. We have sought to explore and understand the cosmos, but they have never been within our reach as much as they are today. The theories we had formed of cosmology have changed dramatically over time. From Ptolemy's geocentric universe to Copernicus' heliocentric universe, and finally, to Einstein's General Relativity and modern cosmology, we have come a long way.

We now have a framework in the form of the big bang theory that explains almost all the observations we have today. Although it was widely disregarded at the time of its conception, because of the anomalous age of the universe the model predicted, it has since only gained evidence in its support, and the revised estimates are much more agreeable.

In this document, I present a review of the current theories of the structure of the universe. It starts with a brief review of the big bang and its consequences. Then we talk about the formation of the universe in the first few minutes and the inflationary period. Finally, we conclude by describing the ΛCDM model that gives the structure of the universe in the present day.

I also discuss some simulation techniques being implemented today to study the how the structure of the universe might have evolved over time to yield the present day scenario.

2 The cosmological principle

2.1 Assumptions

The standard cosmological models are based on the so-called cosmological principle, which is composed of two assumptions. The first assumption is that the cosmos is a manifold $M \times \mathbb{R}$ endowed with a Lorentzian metric, four-dimensional in the usual case, such that the lines $x \times \mathbb{R}$, $x \in M$, called trajectories of the 'fundamental observers', are timelike geodesics orthogonal to the manifold $M \times \{t\}$ at each point $(x, t) \in M \times \mathbb{R}$. The Lorentzian metric of the cosmos can therefore be written as

$$^{(4)}g := -dt^2 + {}^{(3)}g$$

The Riemannian manifold $(M, {}^{(3)}g)$ is the universe at time t. The proper time of fundamental observers is called cosmic time. The second assumption of the cosmological principle is that the universe at each time should look the same in all directions and to any fundamental observer. This assumption is called the Copernican principle—in the name of Copernicus, who deprived us of a central position in the Solar System. We also know that the Sun does not occupy a remarkable place in our galaxy. The mathematical content of the second assumption, modulo the first one, is as follows:

1. Isotropy: the Riemann tensor of the space metric $^{(3)}g$ is at each point x invariant under rotation in the tangent space to M centered at x.

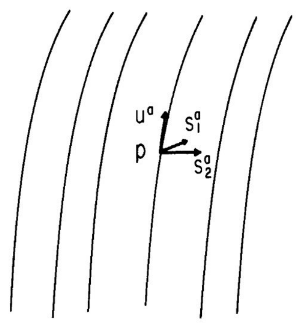


Fig. 5.2. The world lines of isotropic observers in spacetime. By definition of isotropy, for any two vectors s_1^a, s_2^a at p which are orthogonal to u^a, there exists an isometry of the spacetime which leaves p fixed and rotates s_1^a into s_2^a.

2. The space metric is homogeneous, i.e. it admits a transitive group of (global) isometries.

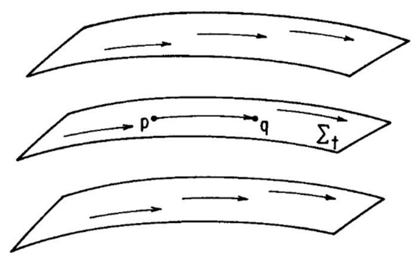


Fig. 5.1. The hypersurfaces of spatial homogeneity in spacetime. By definition of homogeneity, for each t and each $p, q \in \Sigma_t$ there exists an isometry of the spacetime which takes p into q.

2.2 Observational Support

We see the stars very unevenly distributed in the night sky. Galaxies, and even clusters of galaxies, are also observed in our telescopes as being very anisotropically and inhomogeneously distributed. An argument in favour of adopting the cosmological principle is that, at a still larger scale, isotropy and homogeneity seem to be attained. For most astrophysicists, the strongest evidence for the validity of the cosmological principle is the isotropy of the CMB (cosmic microwave background) radiation. This is the faint background glow that sensitive radio telescopes detect in the sky. The CMB radiation is measured to be very nearly isotropic, with a temperature of about 2.725K and a black body spectrum. Keeping aside measurement errors, we also observe anisotropies of the order of 10^{-5}; and the most recently observed are best explained by random inhomogeneities in the big bang.

An argument in favour of homogeneity is that the fundamental physical constants seem to be and have been the same throughout the cosmos with a remarkable accuracy.

3 Robertson-Walker spacetimes

The Robertson–Walker spacetimes are (3 + 1)-dimensional models satisfying the assumptions of the cosmological principle; that is, they are represented by metrics that read, on a product $M \times R$ with M a three-dimensional manifold,

$$^{(4)}g := -dt^2 + {}^{(3)}g$$

where $^{(3)}g$ is a t-dependent Riemannian metric on M whose Riemannian curvature is isotropic at each point of M. We will see that this implies that the metric $^{(3)}g$ is homogeneous.

3.1 Metric at time t

The curvature tensor of a Riemannian metric g at a point $x \in M$ is isotropic at x, that is, invariant under rotations in the tangent space to M at x, if and only if it is of the form

$$R_{ij,hk}(x) = K(x)\big(g_{ih}g_{jk} - g_{jh}g_{ik}\big)(x)$$

The contracted Bianchi identity implies

$$^{(3)}\nabla_j^{(3)} \equiv 0, \text{i.e.}, \partial_i k = 0$$

Hence, K is a constant; the isotropic metric $^{(3)}g$ is also homogeneous. Riemannian spaces with curvature of the form given above with K a constant are called spaces of constant curvature.

To determine the general Riemannian spacetimes (M, *equation here*) of constant curvature, it is convenient to use, in a neighbourhood of an arbitrarily chosen point, polar pseudo-coordinates centred at that point, in which the spherical symmetry resulting from the cosmological principle ismanifest. In these coordinates, the metric takes the form

$$^{(3)}g \equiv e^{\mu}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \qquad \text{with } \mu = \mu(r)$$

The general solution is then given by,

$$e^{-\mu} = 1 - Kr^2$$

We already know that a metric of constant curvature $K = 0$ is locally flat.

A metric with constant curvature $K > 0$ [respectively $K < 0$] is locally a 3-sphere of radius $K^{\frac{1}{2}}$ [respectively locally a hyperbolic 3-space of radius $|K|^{-(\frac{1}{2})}$]. In these cases, one classically scales r by setting $r = |K|^{-\frac{1}{2}}\bar{r}$ and relabels $\bar{r}$ as r. In the new coordinate r, the metric takes one or other of the standard forms according to the sign of K:

$$^{(3)}g \equiv |K|^{-1}\gamma_E \equiv \frac{dr^2}{1-Er^2} + r^2(\sin^2\theta\, d\phi^2 + d\theta^2)\ , \qquad \text{where } \epsilon = \text{sign}(K)$$

3.2 Robertson-Walker cosmologies

Hence, there are three types of Robertson-Walker space-time metrics:

$$-dt^2 + a^2(t)\gamma_\epsilon, \qquad \epsilon = -1, 0 \text{ or } 1$$

Where a is an arbitrary function of t. We see that a(t) is a scaling factor of local spatial distances of fundamental observers. In an expanding universe, (i.e., if $\dot{a} := da/dt > 0$), this distance increases proportionally to a and to the original distance, as in an inflated balloon. Indeed, on the trajectory of a fundamental observer, only t varies; the space distance at time t of two fundamental observers moving respectively on the timelines x = x0 and x = x1 is the product by

a(t) of the distance in the metric γ_ϵ between these two points of M.

4 General properties of Robertson-Walker spacetimes

4.1 Cosmological Redshift

The first essential cosmological data came from the observation of the redshifts of stars and galaxies. The redshift parameter is defined to be

$$z \equiv \frac{\nu_s}{\nu_0} - 1,$$

where ν_0 is the observed frequency and ν_S the emitted frequency. There is a shift towards the red [respectively towards the blue] if $z > 0$ [respectively $z < 0$]. In an expanding Robertson–Walker spacetime, there is a cosmological redshift with parameter given approximately by

$$1 + z \equiv \frac{\nu_s}{\nu_0} \cong \frac{a(t_0)}{a(t_S)},$$

if we assume that the variation of a is negligible during a period of the emitted and a period of the received light signal.

A positive cosmological redshift, $\nu_0 < \nu_S$ signals an expansion of the universe, $a(t_0) > a(t_S)$. Statistical observations of distant galaxies confirm, if we analyze them in the framework of a Robertson-Walker spacetime, that our universe is at present expanding, $a'(t) > 0$. Over the last few years, it has been observed expansion is accelerating: $a''(t) > 0$.

4.2 Hubble law

In a Robertson–Walker spacetime, the distance between two given fundamental observers at some cosmic time t is proportional to a(t). The Hubble parameter is defined to be the constant-in-space, t-dependent scalar

$$H \equiv a^{-1}a'.$$

It measures the rate of expansion (or possibly contraction) of the universe. It has dimension (time)$^{-1}$. An expanding Robertson–Walker universe has a positive Hubble parameter.

The Hubble law says that the observed redshift is proportional to the distance of the source. It is true as consequence of the theory only in first approximation for not-too-distant sources, as wenow show.

Approximate computation of the redshift gives

$$z \cong H(t_0)d_S$$

For a given observer at a given time, the redshift is, in rough approximation, proportional to the distance of the source.

5 ΛCDM cosmological model

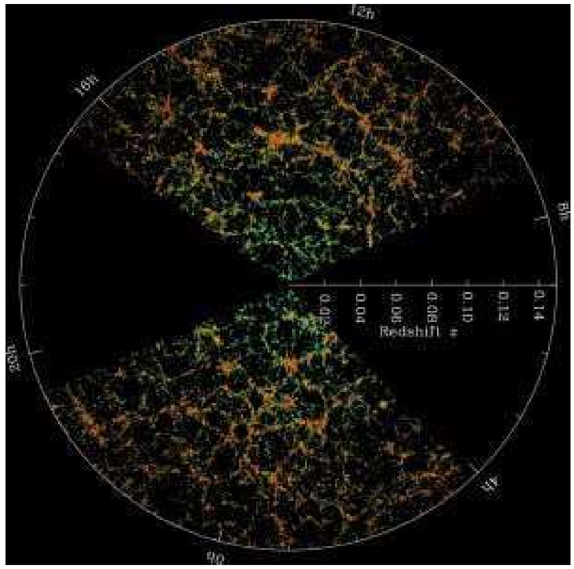
The standard model of the universe adopted at present by most cosmologists is, up to small corrections, a Friedmann–Lemaître universe; that is, a Robertson–Walker spacetime solution of the Einstein equations with cosmological constant and source a stress–energy tensor of matter, with negligible contribution from radiation and neutrinos (even if the latter have a tiny non-zero mass). Observations of the motions of stars and galaxies using powerful Earth-based and satellite telescopes seem to imply that at a cosmological scale classical baryonic matter is a very small part of the energy content of the cosmos, around 4.5%. The observations indicate the existence of cold dark matter, estimated at about 25% of the universe energy content and probably composed of WIMPS (weakly interacting massive patricles) with only weak and gravitational interactions. The acceleration of the universe is explained in the ΛCDM model by the existence of dark energy represented by the term Λg, usually considered as an energy of the vacuum, of quantum origin and constant in spacetime, in agreement with the observational result of the constancy throughout spacetime of the fundamental dimensionless parameters, in particular the fine-structure constant.

6 Large Scale Structure

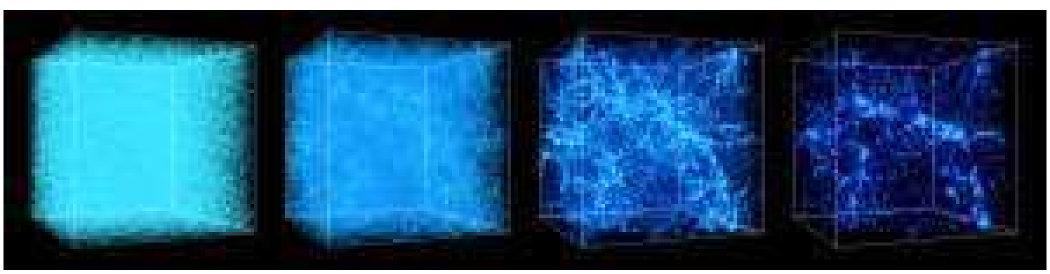
The Large Scale Structure (LSS) of the universe refers to the patterns of galaxies and matter on scales much larger than individual galaxies or groupings of galaxies. These correlated structures can be seen up to billions of light years in length and are created and shaped by gravity. Just as gravity on smaller scales pulls together gas particles to make stars, and pulls together stars to make galaxies, it also pulls together galaxies and matter into patterns on larger scales. These patterns often contain large filaments of galaxies, and voids in between, somewhat resembling a spider web, which is why it is often referred to as 'the cosmic web.'

Studying LSS tells astronomers about the strength of gravity in the universe. Astronomers can measure galaxies at different distances away from the Earth, which correspond to different times in the universe's history, due to the time their light takes to reach us. We can tell that over time, gravity is attracting more and more matter together, clustering the universe further and further.

Large Scale Structure also tells us about dark energy. Most theoretical models of dark energy act to slow down this process of gravity creating large structures. Essentially, as the universe accelerates in its expansion, it takes more time for matter to come together because it must travel more distance. Studying the growth of large scale structure across time gives us information about gravity, dark energy, and how each may be changing as the Universe evolves with time.



This figure shows galaxies discovered by the Sloan Digital Sky Survey (SDSS). Red points are galaxies with more red star light, indicating older and often larger galaxies. The web-like distribution of galaxies on large scales can be seen by eye. A larger redshift corresponds to a larger distance from Earth, which is at center of the figure. The figure shows galaxies out to around 2 billion lightyears away. The Dark Energy Survey will map galaxy positions out to roughly redshift 1.3 or so, around six times as distant as the furthest galaxies seen here. Figure Credit: M. Blanton and SDSS



This figure shows a simulation of gravity in an expanding Universe. As time goes on (left to right), gravity pulls together matter into large scale patterns. Notice that the pattern on the right (present day) has much more clustered structure than the left-most box (early in the Universe). Also, the Universe expands with time, so every 'box' in the Universe is also growing in size. (The expansion has been much bigger since the beginning of the Universe than can be shown here.) These simulations help cosmologists know how gravity is predicted to cluster the Universe over time, which we can compare with our observations. Figure Credit: Andrey Kravtsov, Anatoly Klypin, National Center for Supercomputer Applications (NCSA).

The organization of structure arguably begins at the stellar level, though most cosmologists rarely address astrophysics on that scale. Stars are organized into galaxies, which in turn form galaxy groups, galaxy clusters, superclusters, sheets, walls and filaments, which are separated by immense voids, creating a vast foam-like structure sometimes called the "cosmic web". Prior to 1989, it was commonly assumed that virialized galaxy clusters were the largest structures in existence, and that they were distributed, more or less, uniformly throughout the universe in every direction. However, since the early 1980s, more and more structures have been discovered. In 1983, Adrian Webster identified the Webster LQG, a large quasar group consisting of 5 quasars. The discovery was the first identification of a large-scale structure and has expanded the information about the known grouping of matter in the universe.

6.1 Galaxy

The Milky Way galaxy

About 300,000 years after the big bang, atoms of hydrogen and helium began to form, in an event called recombination. Nearly all the hydrogen was neutral (non-ionized) and readily absorbed light, and no stars had yet formed. As a result, this period has been called the "dark ages". It was from density fluctuations (or anisotropic irregularities) in this primordial matter that larger structures began to appear. As a result, masses of baryonic matter started to condense within cold dark matter halos. These primordial structures would eventually become the galaxies we see today.

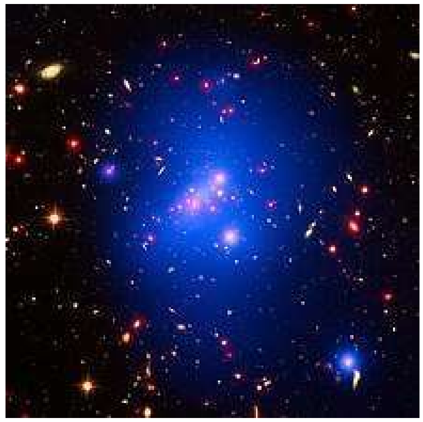
6.2 Galaxy Groups

Four of the seven members of galaxy group HCG 16

Groups of galaxies are the smallest aggregates of galaxies. They typically contain no more than 50 galaxies in a diameter of 1 to 2 megaparsecs (Mpc). Their mass is approximately 1013 solar masses. The spread of velocities for the individual galaxies is about 150 km/s. However, this definition should be used as a guide only, as larger and more massive galaxy systems are sometimes classified as galaxy groups.

Groups are the most common structures of galaxies in the universe, comprising at least 50% of the galaxies in the local universe. Groups have a mass range between those of the very large elliptical galaxies and clusters of galaxies. In the local universe, about half of the groups exhibit diffuse X-ray emissions from their intracluster media. Those that emit X-rays appear to have early-type galaxies as members. The diffuse X-ray emissions come from zones within the inner 10-50% of the groups' virial radius, generally 50-500 kpc.

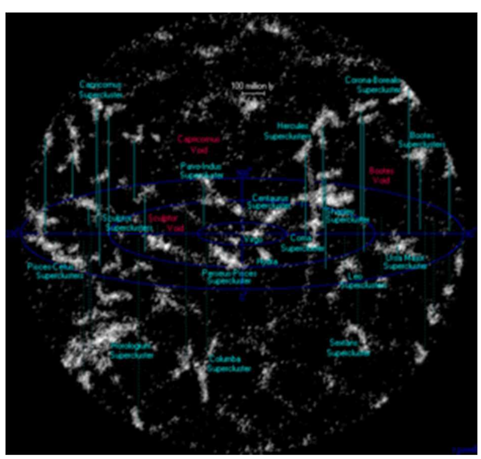
6.3 Galaxy Clusters



Galaxy cluster IDCS J1426 is located 10 billion light-years from Earth and has the mass of almost 500 trillion suns.

A galaxy cluster, or cluster of galaxies, is a structure that consists of anywhere from hundreds to thousands of galaxies that are bound together by gravity with typical masses ranging from 1014-1015 solar masses. They are the largest known gravitationally bound structures in the universe and were believed to be the largest known structures in the universe until the 1980s, when superclusters were discovered. One of the key features of clusters is the intracluster medium (ICM). The ICM consists of heated gas between the galaxies and has a peak temperature between 2–15 keV that is dependent on the total mass of the cluster.

6.4 Supercluster



Map of superclusters closest to Earth

A supercluster is a large group of smaller galaxy clusters or galaxy groups; it is among the largest known structures of the universe. The Milky Way is part of the Local Group galaxy group (which contains more than 54 galaxies), which in turn is part of the Virgo Supercluster, which is part of the Laniakea Supercluster. The large size and low density of superclusters means that they, unlike clusters, expand with the Hubble expansion. The number of superclusters in the observable universe is estimated to be 10 million.

6.5 Galaxy filament

Galaxy filaments are the largest known structures in the universe, consisting of walls of gravitationally bound galaxy superclusters. These massive, thread-like formations can reach 80 megaparsecs h^{-1} (or of the order of 160 to 260 million light-years) and form the boundaries between large voids.

7 Simulation techniques

With the advancement of computer technologies in the last few decades, it has become easier to solve the Friedman equations numerically. Several projects have been carried out to simulate the formation of structure using different cosmology models. Techniques and results from one such simulation is discussed.

7.1 Gravity Calculation and Dark Matter Evolution

Dark matter is represented in cosmological simulations by particles sampling the phase space distribution. Particles are evolved forward in time using Newton's laws written in comoving coordinates

$$\frac{d\vec{x}}{dt} = \frac{1}{a}\vec{v}, \quad \frac{d\vec{v}}{dt} + h\vec{v} = \vec{g}, \quad \vec{\nabla} \cdot \vec{g} = -4\pi G a[\rho(\vec{x}, t) - \bar{\rho}].$$

Here $a(t)$ is the cosmic expansion factor, H is the Hubble parameter, $\vec{v}$ is the peculiar velocity, ρ is the mass density, $\bar{\rho}$ is the spatial mean density, and $\nabla = E\, \partial/\partial\vec{x}$ is the gradient in comoving coordinates. Note that the first pair of relationships is to be integrated for every dark matter particle by using the gravity field produced by all matter (dark and baryonic) contributing to ρ.

The art of N-body simulation lies chiefly in the computational algorithm used to obtain the gravitational force. The desired pair force is a softened inverse square law representing the force between two finite-size particles in order to prevent the formation of unphysical tight binaries. Evaluating the forces by direct summation over all particle pairs is prohibitive even with the largest parallel supercomputers. For collisional N-body systems like globular clusters, where greater accuracy is required, special-purpose processors like the GRAPE series hold great promise.

The hierarchical tree algorithm divides space recursively into a hierarchy of cells, each containing one or more particles. If a cell of size s and distance d (from the point where $\vec{g}$ is to be computed) satisfies $s/d < \theta$, the particles in this cell are treated as one pseudoparticle located at the center of mass of the cell. Computation is saved by replacing the set of particles by a low-order multipole expansion due to the distribution of mass in the cell.

7.2 Gas Dynamics

In comoving coordinates, the cosmological fluid equations are

$$\frac{\partial}{\partial t}\left(\frac{\rho_b}{\bar{\rho}_b}\right) + \frac{1}{a}\vec{\nabla} \cdot \vec{v}_b = 0$$

$$\frac{\partial \vec{v}_b}{\partial t} + \frac{1}{a}\vec{v}_b . \vec{\nabla}\,\vec{v}_b + H\vec{v}_b = -\frac{1}{a\rho_b}\vec{\nabla}p + \vec{g}$$

where $\rho_b, \bar{\rho}_b, \vec{v}_b$, and p are the (baryonic) mass density, mean mass density, peculiar velocity, and pressure, respectively, and $\vec{g}$ is the gravitational field.

Smooth-particle hydrodynamics (SPH) is a Lagrangian (particle-tracking) method for integrating the fluid equations. The fluid variables (baryon density, velocity, temperature, etc) are followed using particles of fixed mass representing fluid elements. The method is therefore an extension of N-body methods, making it relatively easy to add SPH to existing cosmological simulation codes. Because SPH is Lagrangian, the mass continuity equation is obviated. The baryonic mass density is estimated by treating each particle as spread out with a smoothing kernel W:

$$\bar{\rho}_b(\vec{x}) = \sum_{i=1}^{N} m_i W(\vec{x} - \vec{x}_i, h)$$

where m_i and $\vec{x}_i$ are, respectively, the particle mass and position, and h is a smoothing length. A kernel of compact support such as a spline is used so that the sum extends only over particles closer than some cutoff radius proportional to h. These particles are easily found from neighbour lists constructed with the tree or P3M algorithms. The smoothing length generally is taken to vary with $\rho_b^{-1/2}$ so that a fixed number of particles (typically 30–40) is included in the kernel sum.

7.3 Additional physics

Besides gravity and adiabatic gas dynamics, atomic and radiative processes are very important in the formation of galaxies and the evolution of the intergalactic medium. In particular, radiative cooling is thought to be primarily responsible for the condensation and survival of galaxies within larger virialized structures. The processes included in state-of-the-art cosmological simulation codes include optically thin radiative cooling, multispecies chemistry, a phenomenological treatment of star formation and its associated energy feedback, and approximate radiative transfer.

7.4 Initial Conditions

The background model is generally taken to be a spatially flat or open Robertson-Walker spacetime with specified composition of dark matter, baryons, a possible cosmological constant, etc. Specifying such a model requires more than just the two numbers H_0 and Ω (or the deceleration parameter q_0); at the very least the amount and nature of dark matter must be given.

7.5 Results

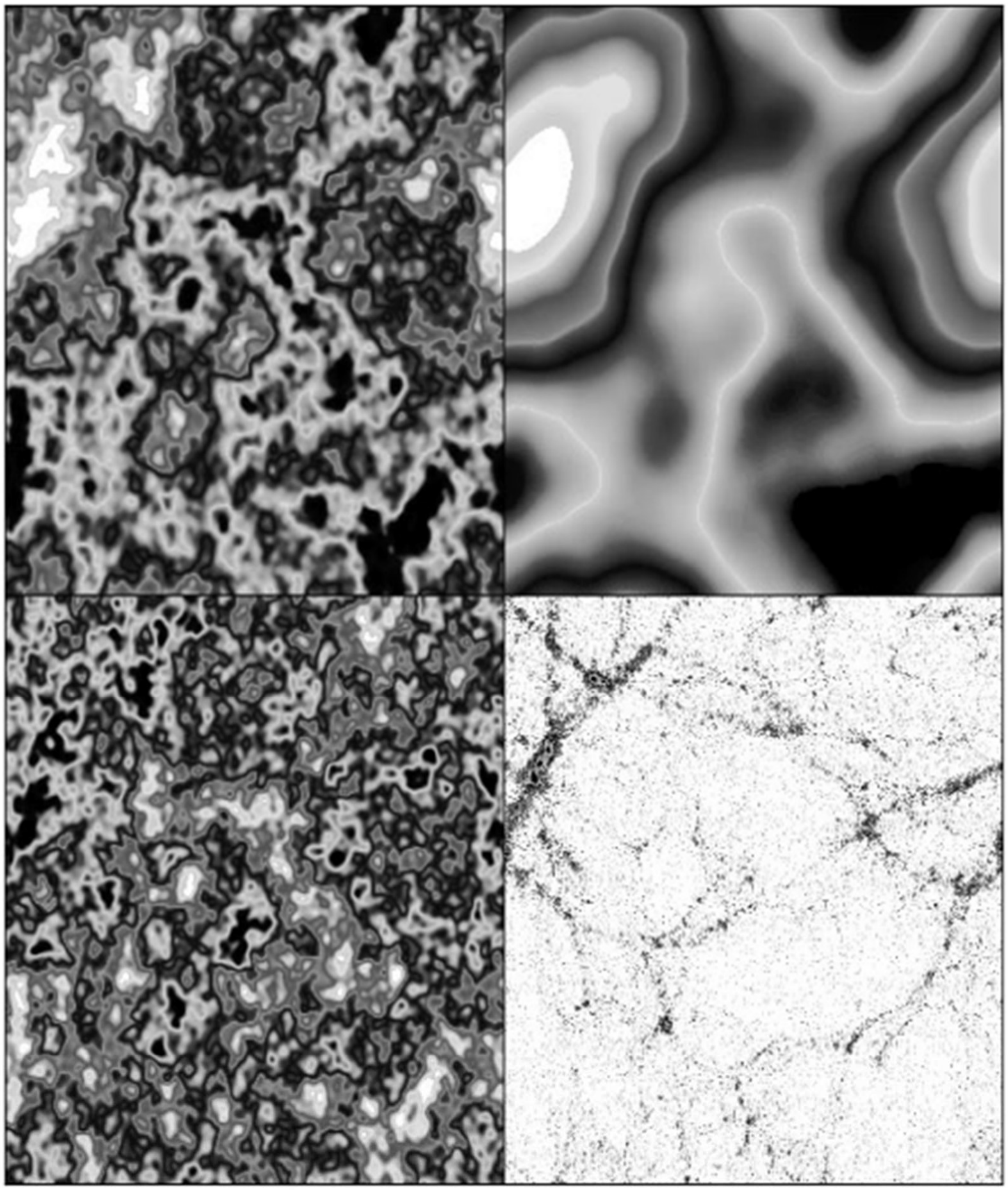


Figure 2 Evolution of the potential and density in a simulation of a hot plus cold dark matter (HCDM) universe in a cube 50 h^{-1} Mpc across. *Top left*: scale-invariant gravitational potential fluctuations in the early universe. *Top right*: Post-recombination potential, showing the modulation by the transfer function. *Bottom left*: Post-recombination density fluctuations. *Bottom right*: Non-linear density field at redshift 0, from a simulation with $\Omega_\nu = 0.2$ by Ma & Bertschinger (1994b).

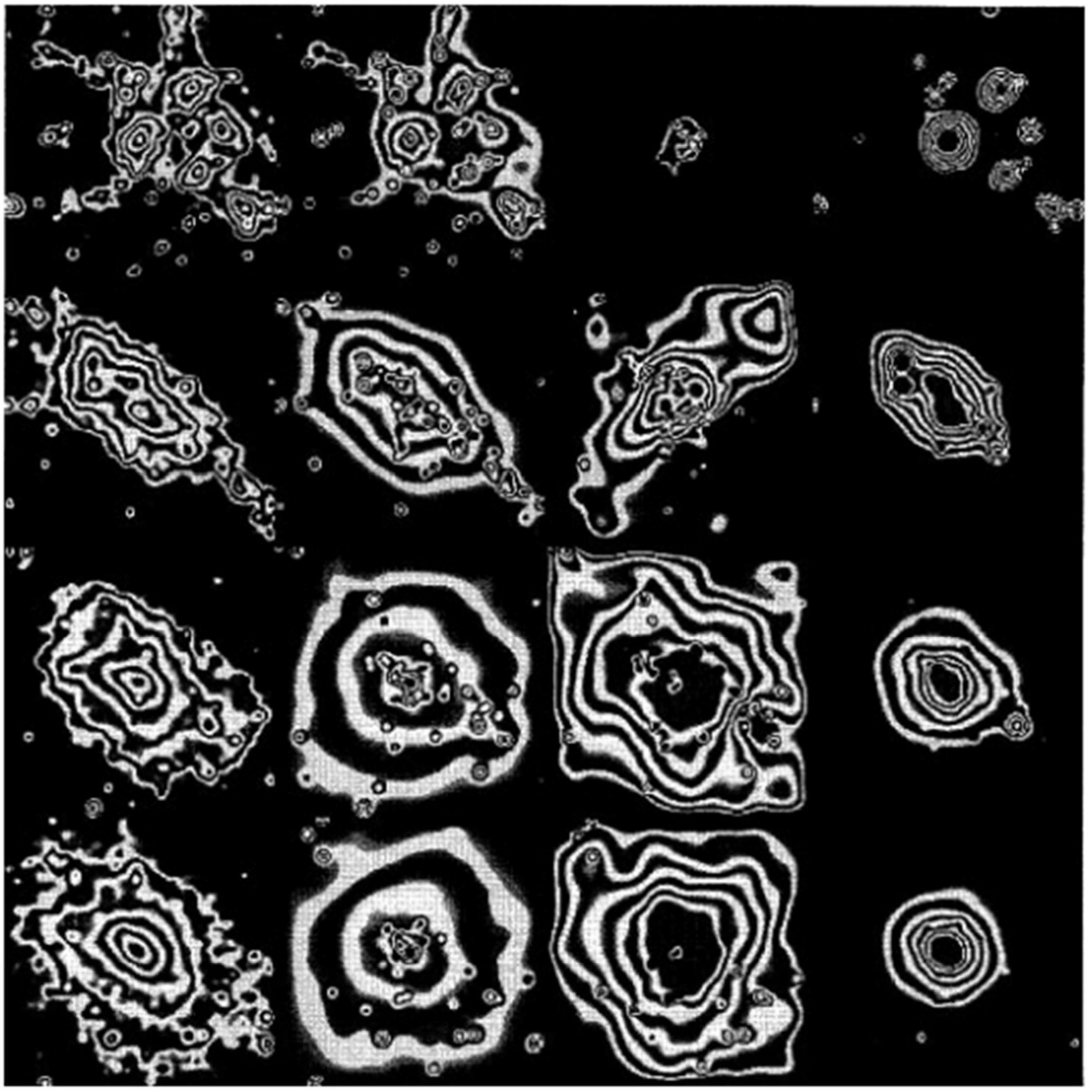


Figure 3 Evolution of an X-ray cluster in the standard cold dark matter (CDM) model. *Columns* from left to right show the projected dark matter density, projected baryon density, emission-weighted temperature, and predicted *ROSAT* X-ray surface brightness. *Rows* from top to bottom show the cluster at redshifts $z = 0.7, 0.3, 0.1$, and 0.03, respectively. From Frenk et al (1996).

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