<u>Research Question</u>

An Investigation into why some Musical Intervals are more Consonant than the others.

<u>By: Devansh Dhabhai</u>
IBDP-I, Neerja Modi School,
Jaipur, India

Table of Contents

I. Introduction

Music is the science of mixing vocal or instrumental (or both) sounds to achieve beauty of shape and harmony; it has always been an important component of human life. It is an inextricable part of the human condition., Like mathematics, music has always been an important part of cultures. Music is a kind of artistic expression that is frequently used to express and portray one's personality and individuality. Music of various genres is studied, performed, played, and listened to.

Music theory is a fascinating subject that has been studied for centuries. The study of the concepts and compositional methods involved in the creation of music is known as music theory. It may include the analysis of any statement, belief or conception of or about music. Music theorists frequently research the language and notation of music. They are looking for patterns and frameworks in composers' practices, as well as across or within genres and historical periods.

When the fundamental de1nitions of mathematics and music are compared, it becomes clear that they are two distinct sciences. Mathematics is a scienti1c discipline that is characterized by order, countability, and calculability. On the other side, music is seen as artistic and expressive. Despite their apparent diIerences, these two disciplines are intertwined and have been for over two thousand years. Music is mathematical in nature, and many basic concepts in music theory are mathematical in nature as well. Experts in diIerent 1elds, including music theorists, use mathematics to develop, express, and explain their ideas.

Many musical phenomena and notions can be described using mathematics. Sound waves are used to express mathematical frequencies, and mathematics explains how strings vibrate at certain frequencies. Cellos have a speci1c form that allows them to resonate with their strings in a mathematical manner. Math is also employed in modern technology to create recordings on a compact disc (CD) or a digital video disc (DVD). As these examples demonstrate, the relationship between mathematics and music is intricate and evolving.

This report aims to give an overview of this intricate relationship between mathematics and music by examining consonance and dissonance. I’ll investigate why some musical intervals are more consonant than others. Being a musician, I’ve always been fascinated by melodies and various harmonies. Hence, to further my understanding on musical consonance and dissonance, I intend to use both the fourier and the harmonic analysis on G, A, and D major chords of a tuned and untuned guitar for 6 seconds.

II. Sound Waves and the Vibration of Strings

Sound is transmitted via a pressure wave within a material. Such a pressure wave can form when an object, vibrating back and forth rapidly, pushes air forward to make way for itself, then moves away again, leaving a partial vacuum behind. A string that is under more tension will vibrate more rapidly, creating pressure waves that are closer together, and hence have a higher frequency. Thicker or longer strings, on the other hand, vibrate more slowly, creating pressure waves that are farther apart, and thus that have a lower frequency. The loudness of a sound corresponds to the amplitude of a pressure wave; the higher the pressure at the peak of the wave, the louder the sound seems to us. The only way to make a string sound louder is to put more energy into it, which you may do by plucking it harder. The physical length of a sound wave Rowing through the air is its wavelength. If you could freeze a sound wave in time and space (and observe it), you could calculate the wavelength by measuring the distance between one peak and the next.

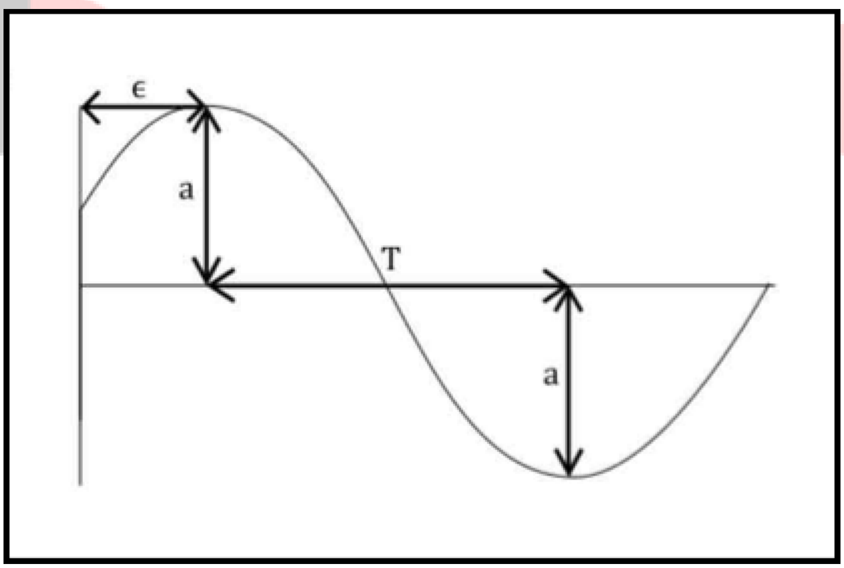

Diagram 1: Graphical Representation of a Sinusoidal Wave

The above diagram represents the most simple sound to which we can refer to as a pitch. The pitch is a simple periodic function of time, represented by a sinusoid. The frequency(f) of a pitch is the number of oscillations it requests per second, measured in Hertz, and the inverse of the frequency, the period(T) is the number of seconds per oscillation. In formulae it's standard

to use rotational frequency(ω) which are given in terms of $\frac{2\pi}{T}$. This means ωt, where t is the

time elapsed in number of radians rotated as 2π is a full turn.The other two properties of pitches are amplitude(a) and phase shift(ϵ). The amplitude is the maximum displacement of the body from equilibrium, and the phase shift- the shift in time from the original wave.

III. Fourier series and Fourier Transform:-

The invention of the mathematical construct presently known as the Fourier transform was driven by two increasingly prevalent and visible di culties in physics. The heat conduction in solids and the motion of a plucked string with 1xed ends are the two issues at hand. Because string instruments like violins and pianos make music by amplifying the vibrations of a 1xed string, the relevance to chord identi1cation is obvious. The creation of the Fourier series and transform has an intriguing and colorful history. The introductions should be read by everyone interested in its progress. Apart from the history, the most important takeaway from the creation of Fourier analysis is that functions may be expressed in both the time and frequency domains.

A periodic system, such as a vibrating string, makes the relationship between the two domains the easiest to see. The motion of a string would most likely be described by focusing on the changing position of spots on the string throughout time. A diIerential equation can be used to model such motion more precisely, with the initial condition being the initial displacement. The velocity of the string is represented by the function $f(t)$ (where t is time) obtained by solving such diIerential equations. It'll provide information on how the string behaves as time progresses.

Now picture plucking the string in such a manner that the string only vibrates at the fundamental harmonic seen in Figure 2. In the time domain, this pattern is represented by a single sinusoid of frequency ν_0. Notice how this frequency ν_0 and the amplitude of the oscillation exactly describe the motion of the string. As a result, the frequency domain $f(v)$ of this particular string contains only one spike at $\nu = \nu_0$, with the height of the spike equal to the wave's amplitude. The fundamental harmonic is a good illustration of how to visualize the frequency domain, but in real systems, there are usually several frequencies.To account for this, one constructs the frequency domain representation by an in1nite series of these harmonics weighted in such a way that they represent the motion of the string. This series is known as the Fourier Series

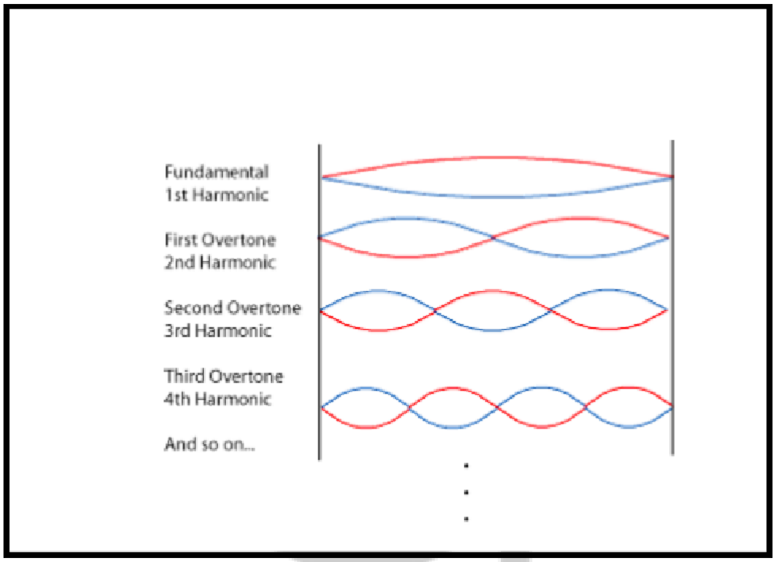

Diagram 2: Graphical Representation of the Harmonics in a Guitar String

Through the Fourier transform one is able to obtain the frequency domain representation of a time domain function. The Fourier transform is invertible with the inverse Fourier transform returning the time domain function from a frequency domain function.

In the time domain, audio signals are captured and listened to; they contain the sound's amplitude as a function of time. Frequencies, on the other hand, represent pitches and thus chords. As a result, the Fourier transform is utilized to turn the time domain input signal into a frequency representation that can be evaluated for pitch intensities. The relationship between the pitches at any particular time is then examined in order to determine the chord being played. However, It's vital to remember that the Fourier transform is a complex-valued transformation; we'll see that the Fourier transform will yield a complex-valued frequency domain representation even for real-valued inputs like audio signals.

IV. Fast Fourier Transform using MATLAB:-

Since my research aims to understand the reasons between consonance and dissonance while playing tuned and untuned guitar chords, I'll be using the Fast Fourier transform to analyze the graphs of the tuned and untuned guitar chords.

A. <u>G Major Chord</u>

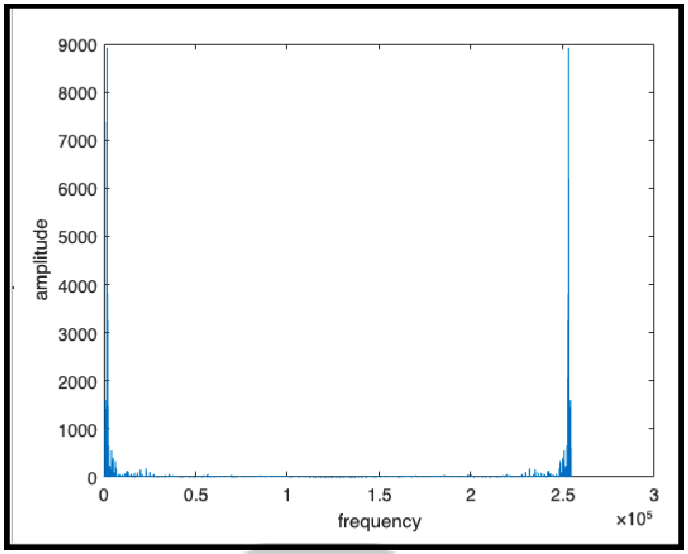

Diagram 3(a): Fourier Transform of Untuned 'G' Chord

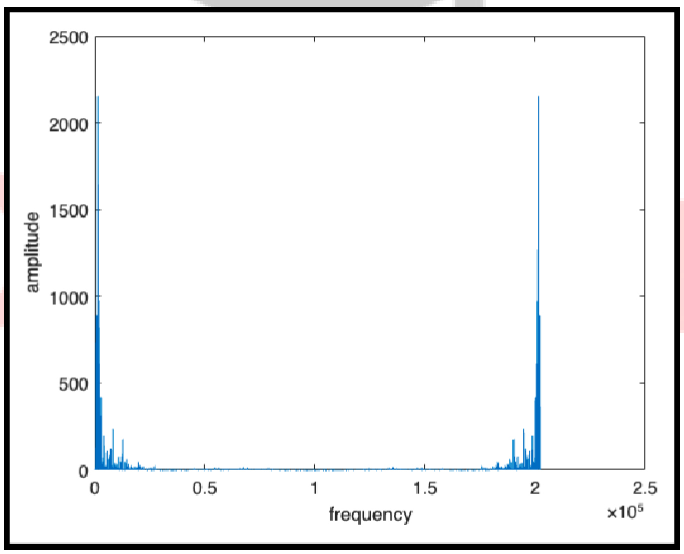

Diagram 3(b): Fourier Transform of Tuned "G" Chord

B. <u>D Major Chord</u>

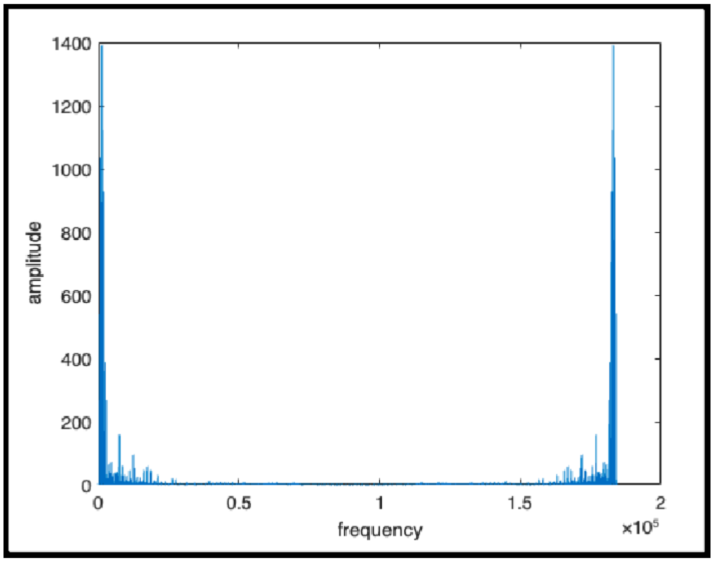

Diagram 4(a): Fourier Transform of Tuned D Major Chord

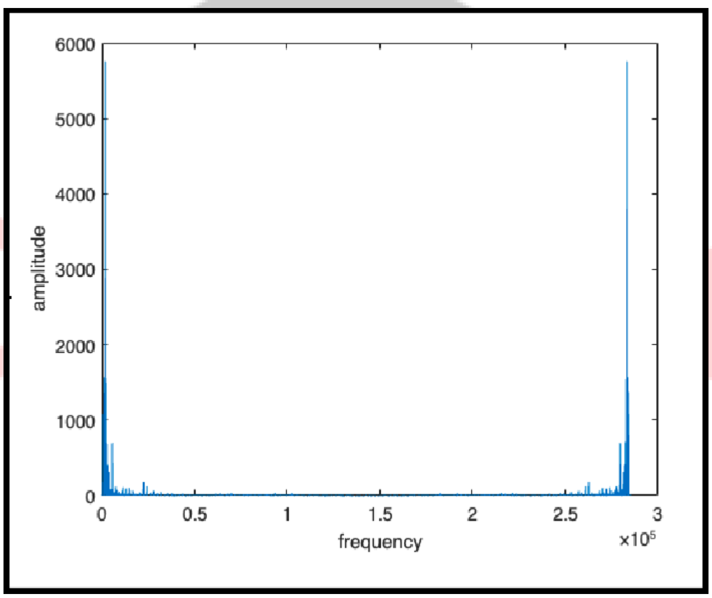

Diagram 4(b): Fourier Transform of Untuned D Major Chord

C. A Major Chord

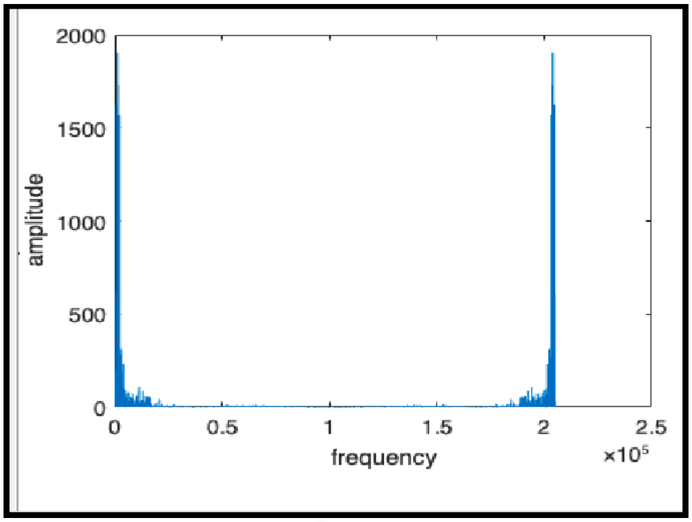

Diagram 5(a): Fourier Transform of Tuned A Major Chord

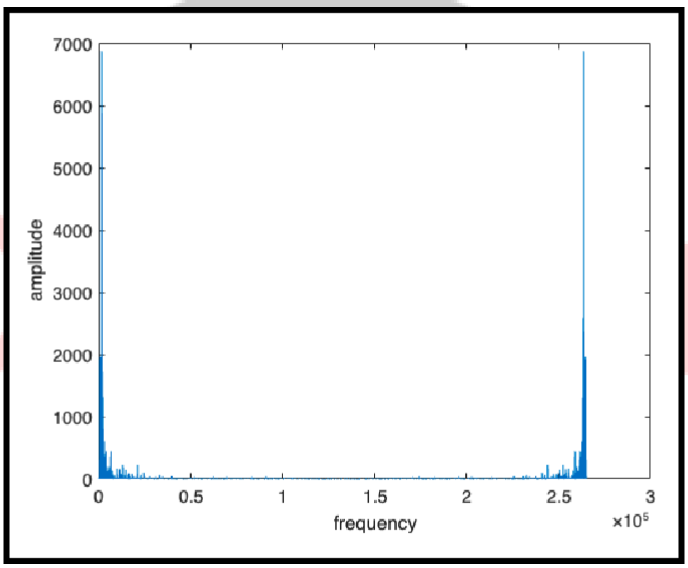

Diagram 5(b): Fourier Transform of Untuned A Major Chord

V. Conclusion

In the above graphs 3(a), 3(b); 4(a), 4(b); 5(a), 5(b) there are primarily two spikes in the frequency: one in the beginning and the other towards the end. Furthermore, we can conclude that the initial frequency is more or less the same accounting for diIerences in the amplitude and observational biases. However, as we move further away from the initial spike the amplitude of the lower spikes in diagram 3(b) are comparatively higher than those formed in diagram 3(a). Therefore, we can infer that these frequencies are more pleasurable to the ear. Additionally, one major diIerence between the two graphs is that the second spike is at diIerent frequencies for both the graphs. For graph 3(a) the second spike is at $2.5 \times 10^5 hz$ while for graph 3(b) the second spike is around $2.0 \times 10^5 hz$. These intricate diIerences between the graphs may suggest why graph 3(a) is consonant in nature.

Moving to graphs 4(a) and 4(b) as mentioned earlier they also follow the same pattern as shown in the other graphs: showing two spikes or in other terms two main frequencies. Particularly, in graph 4(a) the second spike is a little over $1.5 \times 10^5 hz$; however, in graph 4(b) the second spike is over $2.5 \times 10^5 hz$. This data once again validates the fact as to why the D major chord sounded consonant relative to the untuned D major chord.

Finally, the graph 5(a) ,like the other graphs, gives similar types of results. The second spike in graph 5(a) is located around $2.5 \times 10^5 hz$ while in graph 5(b) it's located a little beyond $2.5 \times 10^5 hz$.

VI. Bibliography

Abel, M., Ahnert, K., and Bergweiler, S. (2009). Synchronization of sound sources. *Phys. Rev. Lett.*
103:114301. doi: 10.1103/PhysRevLett.103.114301

Benade, A. H. (1973). The physics of brasses. *Sci. Am.* 229, 24–35. doi: 10.1038/scienti1camerican0773-24

Bidelman, G. M., and Heinz, M. G. (2011). Auditory-nerve responses predict pitch attributes related to musical consonance-dissonance for normal and impaired hearing. *J. Acoust. Soc. Am.* 130, 1488–1502. doi: 10.1121/1.3605559

Bidelman, G. M., and Krishnan, A. (2009). Neural correlates of consonance, dissonance, and the hierarchy of musical pitch in the human brainstem. *J. Neurosci.* 29, 13165–13171. doi: 10.1523/JNEUROSCI.3900-09.2009

Bowling, D. L., Hoeschele, M., Kamraan, Z. G., and Tecumseh Fitch, W. (2017). The nature and nurture of musical consonance. *Music Percept.* 35, 118–121. doi: 10.1525/mp.2017.35.1.118

Bowling, D. L., and Purves, D. (2015). A biological rationale for musical consonance. *Proc. Natl. Acad. Sci. U.S.A.* 112, 11155–11160. doi: 10.1073/pnas.1505768112

Cartwright, J. H. E., Douthettb, J., González, D. L., Krantzd, R., and Piro, O. (2010). Two musical paths to the Farey series and devil's staircase. *J. Math. Music* 4, 57–74. doi: 10.1080/17459737.2010.485001

Cartwright, J. H. E., Gonzalez, D. L., and Piro, O. (2001). Pitch perception: a dynamical-systems perspective. *Proc. Natl. Acad. Sci. U.S.A.*98, 4855–4859. doi: 10.1073/pnas.081070998

Cartwright, J. H. E., Gonzalez, D. L., Piro, O., and Stanziali, D. (2002). Aesthetics, dynamics, and musical scales: a golden connection. *J. New Music Res.* 31, 51–58. doi: 10.1076/jnmr.31.1.51.8099